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ON A PROBLEM OF ISOMERISM.

By F. Franklin, *Fellow of the Johns Hopkins University.*

In the partial discussion of a problem of isomerism here given, use is made of the following theorem:

Write all the partitions of n which do not contain more than one 1; let each partition containing 1 count as one, and let each partition not containing 1 count as the number of *different* numbers which occur in it; the sum of the numbers thus obtained is the number of partitions of $n-1$.

For example: Partitions of 6.

6	1
51	1
42	2
33	1
321	1
222	1

Total, 7, and there are 7 partitions of 5.

The proof is as follows:

A. It is plain, in the first place, that the number of partitions of $n-1$ is equal to the number of partitions of n which contain 1; for all the partitions of n which contain 1 become, by dropping a 1, partitions of $n-1$, and all the partitions of $n-1$ become, by affixing a 1, partitions of n containing 1.

B. Secondly, the number of partitions of n which contain more than one 1 is equal to the number formed by counting the partitions not containing 1 in the manner above prescribed. For every partition not containing 1 can be converted into a partition containing more than one 1 by replacing any number comprised in it by the same number of 1's; and it is easily seen that by doing this for each of the *different* numbers in each partition not containing 1, we obtain all the partitions containing more than one 1, and that we get no repetitions. That we obtain all is evident from the fact that *any* partition containing more than one 1 can be converted into a partition not containing 1 by merging the 1's into a single number. That we get no repetitions is clear when we consider, first, that the *same* partition cannot give two identical results, since in each result we have a different number of 1's; and secondly, that if two identical results arose from two *different* partitions, we could, by merging the 1's in each of these identical results, get back to the two different partitions, which is absurd.

Now, the number of partitions of $n-1$ is equal, as we have seen in A, to the number of partitions of n which contain 1; that is, to the number of partitions of n which contain one 1, *plus* the number of partitions of n which contain more than one 1; and we now see from B that this sum is the number obtained by the rule given at the outset.

The problem of isomerism above alluded to is as follows:

Required to find the number of different compounds that can be formed by n m-valent atoms and $(m-2)\,n+2$ univalent atoms; the word "compound" being understood to mean any arrangement, *whether continuous or not*, in which every atom appears, with exactly the number of bonds to which its valence entitles it; it being understood, moreover, that no two univalent atoms are connected with each other.

The problem reduces at once to the following: In how many ways can $n-1$ bonds be distributed among n atoms, no atom having *more* than m bonds attached to it? (Of course, it is understood that both ends of every bond are attached.) A little consideration would show the truth of this; but it can very easily be formally proved, as follows:

Let x be the number of bonds connecting m-valent atoms with each other; then, since the whole number of bonds, counting those which connect m-valent atoms *twice*, is mn, the number of bonds which connect m-valent atoms with univalent atoms—in other words, the number of univalent atoms—is

$$mn-2x\,.$$

But the number of univalent atoms is required to be $(m-2)\,n+2$. We have, therefore,

$$mn-2x=(m-2)\,n+2\,,$$

whence

$$x=n-1\,.$$

If combinations of univalent atoms were not excluded, the solution would be (as is easily seen from the above) equal to the sum of the numbers of ways in which

$$n-1,\ n,\ n+1,\ \ldots\ldots\ \frac{mn}{2}\ \left(\text{or } \frac{mn-1}{2}\right)$$

bonds can be distributed among n atoms, no atom having more than m bonds attached to it.

When $m=2$, the problem can be easily solved. Let us take it as first stated, *i. e.*, univalent atoms not combining with each other. For $m=2$, it reduces to the following: In how many ways can $n-1$ bonds be distributed among n atoms, no atom having more than 2 bonds attached to it?

Let us consider first, what partitions of n (*i. e.*, what groups of the n atoms) are admissible. Since $n-1$ atoms are *necessary* in order that we should have $n-1$ bonds, it is plain that all partitions which contain 1 more than once must be excluded; for every such partition corresponds to a case in which there occur more isolated atoms than one, *i. e.*, less than $n-1$ atoms furnished with bonds. And every partition in which a single 1 occurs, corresponds to one and only one compound of the kind required. For, by making a *closed circuit* of atoms to correspond to each number greater than 1 in the partition (and only by so doing) we obtain $n-1$ bonds as required. We have now only to look, in addition, at the partitions in which 1 does not occur. In order that the compound represented by such a partition should be of the kind required, we must make a closed circuit to correspond to each number with one exception, one bond being left out in the exception. Now, any one of the numbers entering into the partition can be chosen as the exception; so that the partition will represent, in all, as many compounds as there are *different* numbers in the partition. To sum up the above: we must form all the partitions of n which do not contain more than one 1; let each of these partitions which contains 1 count as one, and let each partition not containing 1 count as the number of different numbers which occur in it; the sum of the numbers thus obtained is the required number of compounds. Now, this sum has been proved to be equal to the number of partitions of $n-1$, so that we have: the number of different compounds that can be formed by n bivalent atoms and 2 univalent atoms is equal to *the number of partitions of* $n-1$, if the union of the univalent atoms is not allowed.

If the union of the univalent atoms had been allowed, our result would have been *the number of partitions of n.* For we would then have had to find the number of ways in which n bonds can be distributed among n atoms, and add this to the above. It is obvious that this number is the number of partitions of n not containing 1; for if we had an isolated atom we could not have n bonds, so that we can have no partitions containing 1; and for any partition not containing 1, by making a closed circuit of atoms to correspond to each number in it (and in no other way) we obtain n bonds, as required. Now, from A, the number of partitions of n not containing 1 is the excess of the whole number of partitions of n over the whole number of partitions of $n-1$. Hence, adding the number of partitions of n not containing 1 to the whole number of partitions of $n-1$, we obtain for our result the whole number of partitions of n.

NOTE ON INDETERMINATE EXPONENTIAL FORMS.

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IF y and z are two functions of x, which become each equal to 0 for a particular value of x, y^z has, for that value, an indeterminate form; and its value is obtained through that of its logarithm. But we shall see that it is generally unnecessary to specially investigate the expression, its value being always 1, provided that the ratio of z, or of some finite power of z, to y, is not infinite.

For we know that the value of x^x when $x=0$ is 1; hence we have $y^z = (y^y)^{\frac{z}{y}} = 1$, provided $\frac{z}{y}$ is not infinite. If $\frac{z}{y}$ is infinite, suppose $\frac{z^k}{y} = a$, where a is finite; then we have

$$y^z = \left(\frac{z^k}{a}\right)^z = \frac{z^{kz}}{a^z} = \frac{(z^z)^k}{a^z} = 1,$$

unless k is infinite. That is, the value of the expression is always unity, provided that the ratio of z, or of some finite power of z, to y, is not infinite. This condition is, I believe, always fulfilled except in some cases when y or z is itself an exponential or logarithmic function; it is, at any rate, generally easy to see at a glance whether or not such is the case.

It follows from the above that the expression ∞^0 is also always equal to unity, with a similar restriction. For, if y^z assumes this form for a particular value of x, put $y = \frac{1}{u}$ and we have $y^z = \left(\frac{1}{u}\right)^z = \frac{1}{u^z}$; and the value of this expression is, by the preceding, always 1, provided that the ratio of z, or of some finite power of z, to u, is not infinite; or, in other words, provided that the product of z, or of some finite power of z, and y, is not infinite.

The examples given in most of the text-books I have seen, come within the above restriction; the following, for instance, are all the examples of the above forms given by Williamson: ex. 1, p. 102: $(\sin x)^{\tan x}$ when $x = 0$. ex. 2, p. 103: $\left(1 + \frac{a}{x}\right)^x$ when $x = 0$; ex. 3, p. 103: $\left(\frac{1}{x}\right)^{\tan x}$ when $x = 0$; ex; 30, p. 108: $\left(\frac{\log x}{x}\right)^{\frac{1}{x}}$ when $x = \infty$; ex. 44, p. 109: $(\sin x)^{\sin x}$ when $x = 0$. In all these cases we at once recognize that the condition above found is fulfilled, and the value of the function consequently 1.

In the following examples, found in Spitz's "Differential- und Integralrechnung," the condition is not fulfilled, and the expressions require a special investigation: $x^{\frac{1}{1 + 2\log x}}$ when $x = 0$; $x^{\frac{1}{\log (e^x - 1)}}$ when $x = 0$; $\left(\frac{1}{1 - e^x}\right)^{\frac{1}{x}}$ when $x = \infty$.

It may be remarked, in this connection, that if y^z assumes, for a particular value of x, the form 1^∞, its value can be very simply expressed without taking its logarithm. Put $z = \frac{1}{u}$; the value required is the limiting value of $(1 + \Delta y)^{\frac{1}{\Delta u}} = \lim\, (1 + \Delta y)^{\frac{1}{\Delta y}\frac{\Delta y}{\Delta u}} = \lim\, \left[(1 + \Delta y)^{\frac{1}{\Delta y}}\right]^{\frac{\Delta y}{\Delta u}} = e^{\frac{dy}{du}}$.